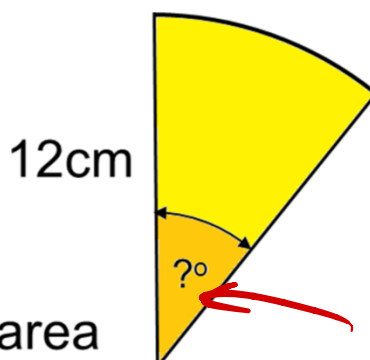
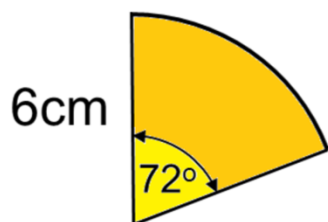


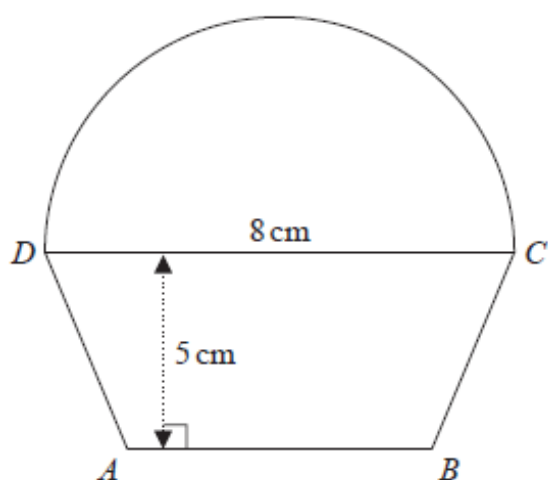
Shape and Space III (3) Further problem solving

Do now:



two sectors have the same area
what is the missing angle?

- 1 The diagram shows a shape made from a trapezium $ABCD$ and a semicircle with diameter DC .



$$\frac{1}{2}\pi\left(\frac{8}{2}\right)^2 + \frac{1}{2}(8+AB)5 = 64$$

$$16\pi + 40 + 5AB = 128$$

$$5AB = 128 - 40 - 16\pi$$

$$AB = \frac{88 - 16\pi}{5}$$

$$= 7.5 \text{ cm}$$

$DC = 8 \text{ cm}$.

The shape has area 64 cm^2

The height of the trapezium is 5 cm .

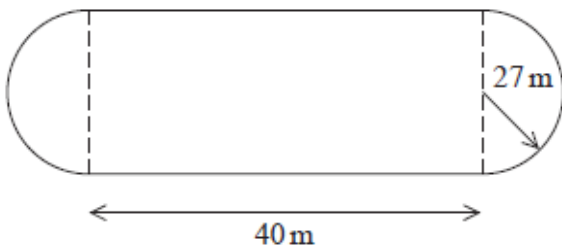
Work out the length of AB .

Give your answer correct to 1 decimal place.

..... cm

(Total for Question 1 is 5 marks)

- 2 The diagram shows a cycle track.



The track has two straight sides each of length 40 m.
Each end of the track is a semicircle of radius 27 m.

The diameter of each wheel of Ian's bike is 590 mm.
Ian is going to ride his bike around the track once.

Calculate how many complete revolutions each wheel of his bike will make.

$$\text{Perimeter} = 249.646$$

$$\text{Circumference} = 0.59\pi$$

$$\text{Revolutions} = \frac{249.64}{0.59\pi}$$

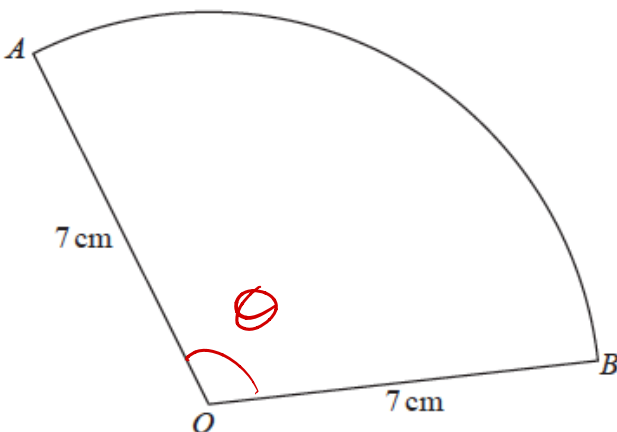
$$= 134.68$$

b
o o

134 complete
revolutions

.....
(Total for Question 2 is 5 marks)

- 3 OAB is a sector of a circle with centre O and radius 7 cm.



The area of the sector is 40 cm^2
Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

$$\frac{\theta}{360} \pi (7^2) = 40$$

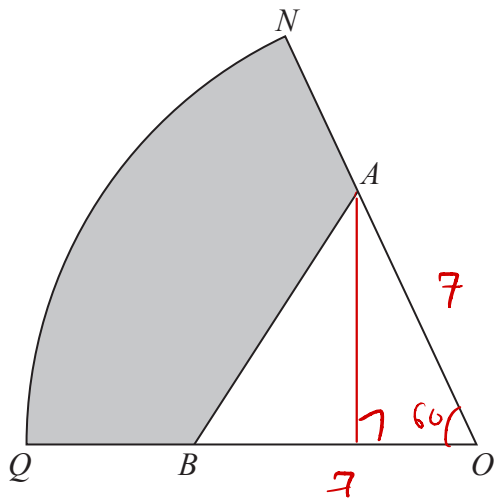
$$\theta = 93.54^\circ$$

$$P = 14 + \frac{93.54}{360} (2\pi \times 7)$$

$$= 25.4 \text{ cm}$$

..... cm
(Total for Question 3 is 4 marks)

4



$$\sin 60 = \frac{h}{7}$$

$$h = \frac{7\sqrt{3}}{2}$$

$$\begin{aligned} \text{Area}_{\Delta} &= \frac{7\sqrt{3}}{2} \times 7 \times \frac{1}{2} \\ &= \frac{49\sqrt{3}}{4} \end{aligned}$$

$$\begin{aligned} \text{SHAD60 AREA} &= \frac{60}{360} \times \pi(11^2) - \frac{49\sqrt{3}}{4} \\ &= 42.1... \end{aligned}$$

ONQ is a sector of a circle with centre O and radius 11 cm.

A is the point on ON and B is the point on OQ such that AOB is an equilateral triangle of side 7 cm.

Calculate the area of the shaded region as a percentage of the area of the sector ONQ.

Give your answer correct to 1 decimal place.

$$\begin{aligned} \% &= \frac{42.1...}{\frac{121\pi}{6}} \times 100 = 66.5\% \end{aligned}$$

SHAD60
TOTAL SECTOR

.....%

(Total for Question 4 is 5 marks)

5. A square, with sides of length x cm, is inside a circle.

Each vertex of the square is on the circumference of the circle.

The area of the circle is 49 cm^2 .

Work out the value of x .

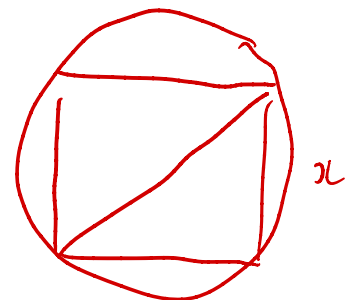
Give your answer correct to 3 significant figures.

$$d = \sqrt{x^2 + x^2} = x\sqrt{2}$$

$$49 = \pi \left(\frac{x\sqrt{2}}{2} \right)^2$$

$$49 = \frac{2\pi x^2}{4}$$

$$x = 5.59$$



(Total for Question 5 is 4 marks)

- 6 The circumference of circle B is 90% of the circumference of circle A.
(a) Find the ratio of the area of circle A to the area of circle B.

$$\begin{array}{ccc} A : B & \xrightarrow{\square^2} & A : B \\ 100 : 90 & & 100 : 81 \\ 10 : 9 & & \end{array}$$

(2)

Square E has sides of length e cm.

Square F has sides of length f cm.

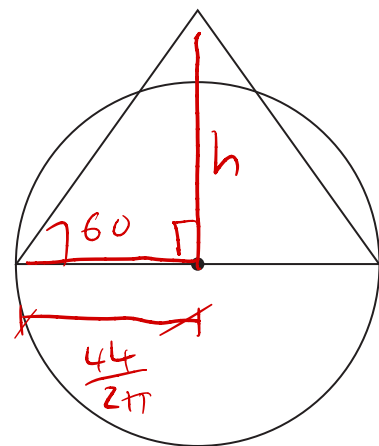
The area of square E is 44% greater than the area of square F.

- (b) Work out the ratio $e : f$

$$\begin{array}{ccc} \text{AREA} & \xrightarrow{\sqrt{}} & \text{LENGTH} \\ E : F & & e : f \\ 144 : 100 & & 12 : 10 \\ & & 6 : 5 \end{array}$$

(2)

- 7 The diagram shows a circle and an equilateral triangle. One side of the equilateral triangle is a diameter of the circle. The circle has a circumference of 44 cm. Work out the area of the triangle. Give your answer correct to 3 significant figures.



triangle.
circle.

$$r = \frac{44}{2\pi}$$

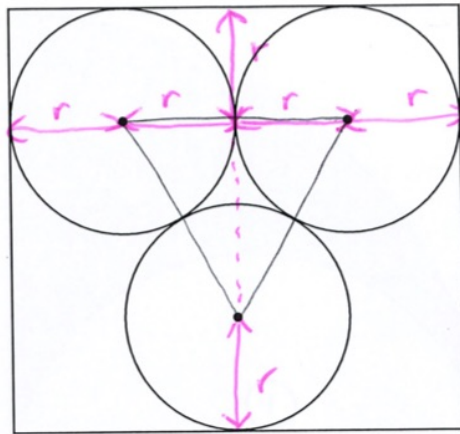
$$h = \tan(60) \times \frac{44}{2\pi}$$

$$\text{AREA} = \frac{44}{\pi} \times \tan(60) \times \frac{44}{2\pi} \times \frac{1}{2} = 84.9 \text{ cm}^2$$

.....cm²
(Total for Question 7 is 3 marks)

8

The diagram shows 3 identical circles inside a rectangle.
Each circle touches the other two circles and the sides of the rectangle, as shown in the diagram.

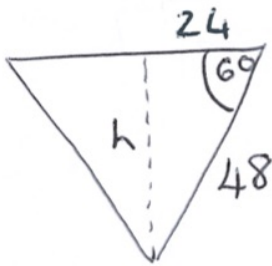


The radius of each circle is 24 mm.

Work out the area of the rectangle.

Give your answer correct to 3 significant figures.

$$\text{Length of rectangle} = 4 \times 24 = 96$$



$$h^2 = 48^2 - 24^2$$

$$h = 24\sqrt{3}$$

$$\text{Height of rectangle} = 24\sqrt{3} + 2 \times 24 = 89.569\dots$$

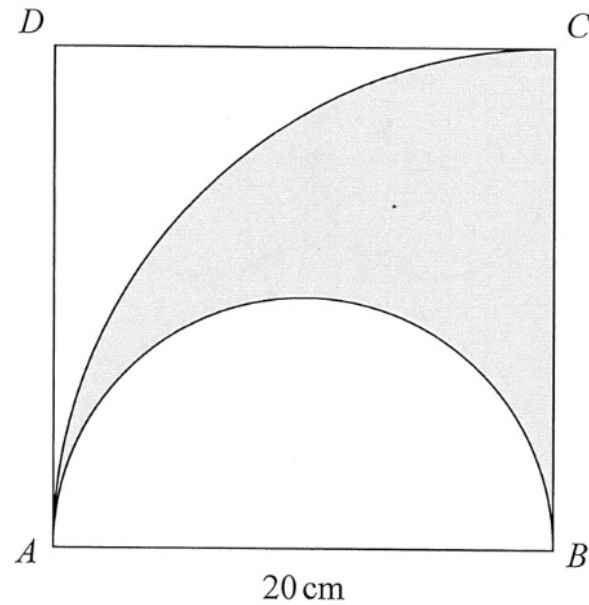
$$\text{Area} = 96 \times 89.569\dots$$

$$= 8600 \quad (3 \text{ s.f.})$$

..... mm²
(Total for Question 8 is 4 marks)

9.

The diagram shows a square $ABCD$ with sides of length 20 cm. It also shows a semicircle and an arc of a circle.



AB is the diameter of the semicircle.
 AC is an arc of a circle with centre B .

Show that $\frac{\text{area of shaded region}}{\text{area of square}} = \frac{\pi}{8}$

$$\frac{\pi \times 10^2}{2} = 50\pi.$$

$$\frac{\pi \times 20^2}{4} = 100\pi.$$

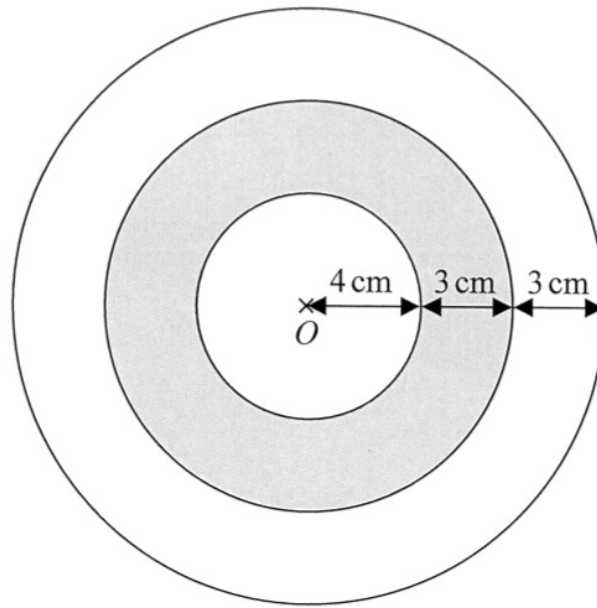
$$\text{Area of square} = 20 \times 20 = 400$$

$$\frac{100\pi - 50\pi}{400} = \frac{\pi}{8}.$$

(Total for Question 9 is 4 marks)

10.

The diagram shows a logo made from three circles.



Each circle has centre O .

Daisy says that exactly $\frac{1}{3}$ of the logo is shaded.

Is Daisy correct?

You must show all your working.

$$\pi \times 4^2 = 50.265 \dots$$

$$\pi \times 7^2 = 153.938 \dots$$

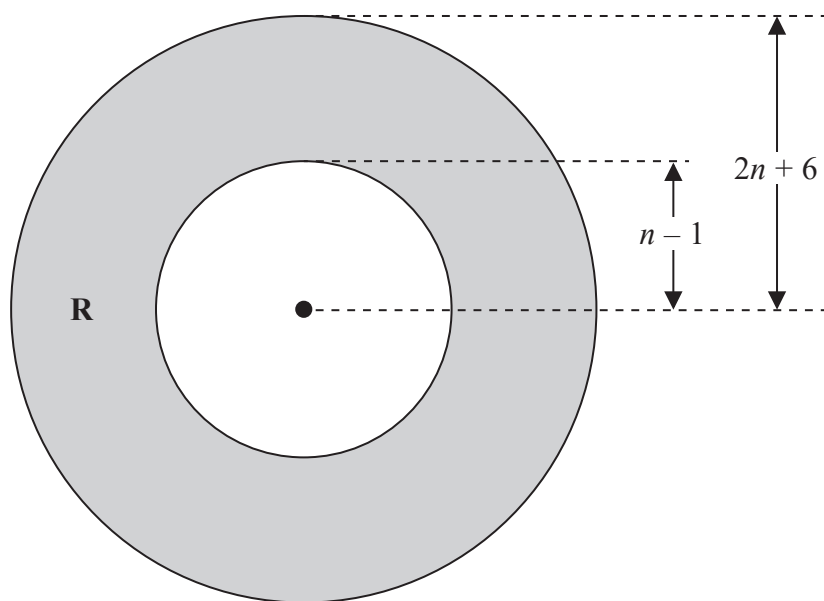
$$\pi \times 10^2 = 314.159 \dots$$

$$\pi \times 7^2 - \pi \times 4^2 = 33\pi.$$

$$\frac{33\pi}{100\pi} = 33\% \neq \frac{1}{3} \quad \text{so Daisy is wrong.}$$

(Total for Question 11 is 4 marks)

- 12** The region **R**, shown shaded in the diagram, is the region between two circles with the same centre.



The outer circle has radius $(2n + 6)$

The inner circle has radius $(n - 1)$

All measurements are in centimetres.

The area of **R** is greater than the area of a circle of radius $(n + 13)$ cm.

n is an integer.

Find the least possible value of n .

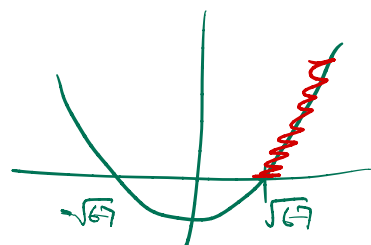
You must show all of your working.

$$\begin{aligned} \underline{\underline{R}} \quad \pi((2n+6)^2 - (n-1)^2) &= 4n^2 + 24n + 36 - (n^2 - 2n + 1) \\ &= 3n^2 + 26n + 35 \end{aligned}$$

$$\begin{aligned} \pi(3n^2 + 26n + 35) &> \pi(n+13)^2 \\ 3n^2 + 26n + 35 &> n^2 + 26n + 169 \end{aligned}$$

$$\begin{aligned} 2n^2 - 134 &> 0 \\ n^2 &> 67 \end{aligned}$$

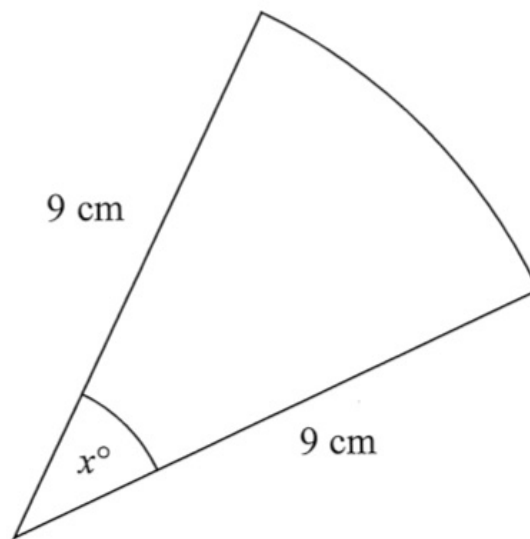
$$\underline{\underline{n = 9}}$$



(Total for Question 12 is 5 marks)

13.

The diagram shows a sector of a circle of radius 9 cm.



The sector has a perimeter of 25 cm.

Work out the value of x .

Give your answer correct to 1 decimal place.

$$25 - 9 - 9 = 7$$

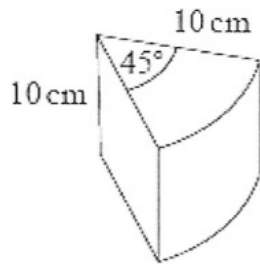
$$\frac{x}{360} \times 2 \times \pi \times 9 = 7$$

$$x = 44.6^\circ$$

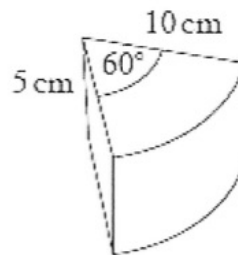
.....
(Total for Question 14 is 4 marks)

16 Here are two solid prisms, prism A and prism B.

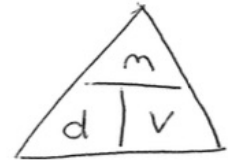
Here are two solid prisms, prism A and prism B.



prism A



prism B



The cross section of prism A is a sector, with angle 45° , of a circle of radius 10 cm. The prism has a depth of 10 cm and a mass of 40π grams.

The cross section of prism B is a sector, with angle 60° , of a circle of radius 10 cm. The prism has a depth of 5 cm and a mass of 50π grams.

Express the difference in the densities of the two prisms as a percentage of the density of prism A.

$$\text{Prism A} = \frac{45}{360} \times \pi \times 10^2 \times 10 = \frac{1000\pi}{8}$$

$$\text{Prism B} = \frac{60}{360} \times \pi \times 10^2 \times 5 = \frac{500\pi}{6}$$

$$\begin{aligned} \text{Density Prism A} &= 40\pi \div \frac{1000\pi}{8} \\ &= 40\cancel{\pi} \times \frac{8}{1000\cancel{\pi}} = 0.32 \end{aligned}$$

$$\text{Density Prism B} = 50\pi \div \frac{500\pi}{6} = 0.6$$

$$\frac{0.6 - 0.32}{0.32} \times 100 = 87.5\%$$

..... %

(Total for Question 10 is 5 marks)